A few useful relations for fluidics

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Einstein relation (for sphere, radius *a*):

$$D = \frac{k_B T}{f} = \frac{k_B T}{6\pi\eta a}$$

At room temperature we have

$$k_B T = 4\,10^{-21} J$$
$$\eta = 10^{-3}\, kg\, m^{-1}\, s^{-1}$$

$$D = \frac{k_B T}{f} = \frac{k_B T}{6\pi\eta a} = 2.12\cdot 10^{-13}\ m^2 s^{-1}\frac{1\mu m}{a}$$

For n-dimensional diffusion we have $\langle r^2 \rangle = 2nDt$

Object	Diffusion coefficient [m^2s^{-1}]	Time to diffuse in 1D		
		1µm	**1 mm**	**1 m**
Water molecule	2.4 10-9	0.2 ms	200 s	7 yrs
10nm radius bead	2.1 10-11	20 ms	6 hrs	700 yrs
100nm radius bead	2.1 10-12	0.2 s	60 hrs	7000 yrs
1µm radius bead	2.1 10-13	2 s	1 month	70000 yrs

Pressure driven flow

$$Q = \frac{\Delta p}{R}, \quad v = \frac{\Delta p}{A\,R}$$

Flow resistance for a shallow slot with w>>h

$$R = \frac{12\eta L}{wh^3}$$

Flow resistance for a cylinder

$$R = \frac{8\eta L}{\pi r^4}$$

So that for a cylinder and a shallow slot the velocities are

$$v_{cylinder} = \frac{\Delta p}{8\eta L} r^2, \quad v_{slot} = \frac{\Delta p}{12\eta L} h^2$$

A few examples for water at room temperature T=25°C in a cylinder that is L=1mm long.

Radius	VELOCITY [length/time]			FLOW [volume/time]		
	10mBar	**100mBar**	**1Bar**	**10mBar**	**100mBar**	**1Bar**
10nm	10nm/s	100nm/s	1µm/s	0.2 aL/min	2 aL/min	20 aL/min
100nm	1µm/s	10µm/s	100µm/s	2 fL/min	20 fL/min	0.2 pL/min
1µm	100µm/s	1mm/s	10mm/s	20 pL/min	0.2 nL/min	2 nL/min
10µm	10mm/s	100mm/s	1m/s	0.2 µL/min	2 µL/min	20µL/min
100µm	1m/s	10m/s	100m/s	2 mL/min	20mL/min	0.2 L/min

The Reynolds number is the ratio between inertial forces and viscous forces.

$$\mathrm{Re} = \frac{v\rho L}{\eta}$$

The Reynolds for water at room temperature T=25°C

Velocity	**Channel diameter**			
	1µm	**10µm**	**100µm**	**1mm**
100µm/s	10^{-4}	10^{-3}	0.01	0.1
1mm/s	10^{-3}	0.01	0.1	1
10mm/s	0.01	0.1	1	10
100mm/s	0.1	1	10	100
1m/s	1	10	100	1000

Inertia dominates for Re >> 1 and viscosity dominates for Re << 1

As of rule of thumb we have the following, although the exact limits are debated:
Laminar flow for Re < 30
Turbolent flow for Re > 2000